Solutions to RSPL/1 (DS1)

1. (*d*) Let $p(x) = 2x^2 + 7x + 5 \Rightarrow a = 2, b = 7$ and $c = 5$

$$\alpha + \beta = \frac{-b}{a} = \frac{-7}{2}$$

$$\alpha\beta = \frac{c}{a} = \frac{5}{2}$$

$$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta} = \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

$$= \frac{\left(\frac{-7}{2}\right)^3 - 3 \times \frac{5}{2}\left(\frac{-7}{2}\right)}{\frac{5}{2}}$$

$$= \frac{\frac{-343}{8} + \frac{105}{4}}{\frac{5}{2}}$$

$$= \frac{-343 + 210}{8} \times \frac{2}{5} = \frac{-133}{20} = -6\frac{13}{20}$$

$\therefore$ Option (*d*) is correct.

2. (*c*) The given equations are

$$m - n = 3 \qquad ...(i)$$

$$\frac{m}{3} + \frac{n}{2} = 6 \qquad ...(ii)$$

From (*i*) we have $m = 3 + n$ ...(*iii*)

Substituting in (*ii*)

$$\Rightarrow \quad \frac{3+n}{3} + \frac{n}{2} = 6$$

$$\Rightarrow \quad 2(3 + n) + 3n = 36$$

$$\Rightarrow \quad 6 + 2n + 3n = 36$$

$$\Rightarrow \quad 5n = 30$$

$$n = 6$$

Substituting in (*iii*)

$$m = 3 + 6$$

$$m = 9$$

$\therefore$ Option (*c*) is correct.

3. (*a*) The given equation is $(k + 4)x^2 + (k + 1)x + 1 = 0$

For equal roots, $b^2 - 4ac = 0$

$$\Rightarrow \quad (k + 1)^2 - 4 \times (k + 4) \times 1 = 0$$

$$\Rightarrow \quad k^2 + 1 + 2k - 4k - 16 = 0$$

$$\Rightarrow \quad k^2 - 2k - 15 = 0$$
$$\Rightarrow \quad k^2 - 5k + 3k - 15 = 0$$
$$\Rightarrow \quad k(k-5) + 3(k-5) = 0$$
$$\Rightarrow \quad (k-5)(k+3) = 0$$
$$\Rightarrow \quad k = 5 \text{ or } k = -3$$

$\therefore$ Option (a) is correct.

4. (c) $p^2 + 4p + 8$, $2p^2 + 3p + 6$ and $3p^2 + 4p + 14$ are three consecutive terms of an AP.

$$\therefore \quad 2(2p^2 + 3p + 6) = p^2 + 4p + 8 + 3p^2 + 4p + 14$$
$$\Rightarrow \quad 4p^2 + 6p + 12 = 4p^2 + 8p + 22$$
$$\Rightarrow \quad 2p = -10$$
$$p = -5$$

$\therefore$ Option (c) is correct.

5. (b) Total number of cards in the pack = 52,

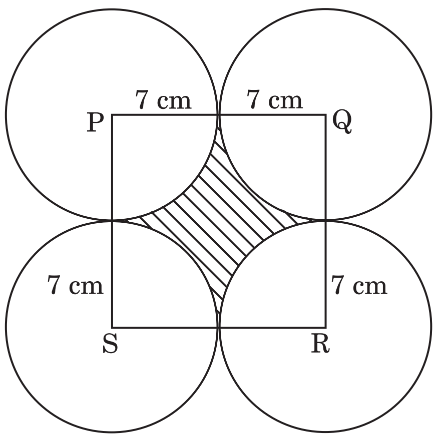
Number of red face cards = 6

$\therefore$ Number of favourable outcomes = 6

$\therefore$ Probability of red face card = $\frac{6}{52} = \frac{3}{26}$

$\therefore$ Option (b) is correct.

6. (d) Area of square PQRS = $(14)^2 = 196$ cm^2



Area of 4 Quadrants = Area of One circle of radius 7 cm.

$$= \frac{22}{7} \times 7 \times 7 = 154 \text{ cm}^2$$

Area of shaded region = $(196 - 154)$ cm^2 = 42 cm^2

$\therefore$ Option (d) is correct.

7. (b) Here radius (r) = 14 cm

Angle at the centre (θ) swept by minute hand in 15 minutes = $\frac{360° \times 15}{60} = 90°$

$\therefore$ Area of the sector formed by minute hand = $\frac{\pi r^2 \theta}{360°} = \frac{22}{7} \times 14 \times 14 \times \frac{90}{360}$ cm^2

$$= 154 \text{ cm}^2$$

$\therefore$ Option (b) is correct.

8. (*b*) We have, $\sin\theta = \cos(2\theta + 45°)$

$\Rightarrow \sin\theta = \sin[90 - (2\theta + 45°)]$

$\Rightarrow \sin\theta = \sin[45° - 2\theta]$

$\Rightarrow \theta = 45° - 2\theta$

$\Rightarrow 3\theta = 45°$

$\Rightarrow \theta = 15°$

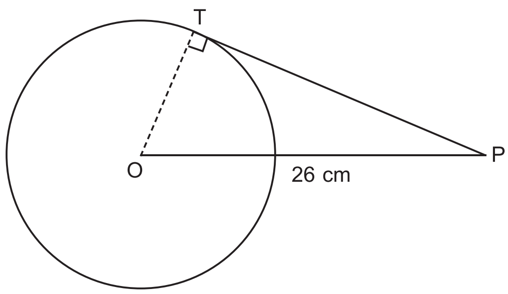
$\Rightarrow \sqrt{2}\sin(3 \times 15°) - 2\tan(3 \times 15°)$

$$= \sqrt{2}\sin 45° - 2\tan 45°$$

$$= \sqrt{2} \times \frac{1}{\sqrt{2}} - 2 \times 1 = 1 - 2 = -1$$

$\therefore$ Option (*b*) is correct.

9. (*b*) Point P is 26 cm from the centre O of the circle



$\therefore \quad OP = 26$ cm

PT is tangent to the circle

$\therefore \quad PT = 24$ cm.

as radius is perpendicular to tangent at point of contact.

In rt. ΔOPT, $\angle T = 90°$

$\therefore \quad OP^2 = OT^2 + PT^2$

$(26)^2 = OT^2 + (24)^2$

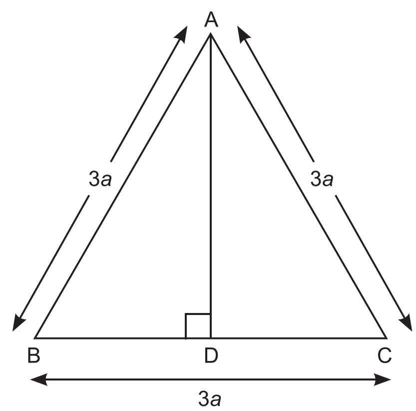
$(26)^2 - (24)^2 = OT^2$

$\Rightarrow \quad OT = \sqrt{676 - 576} = \sqrt{100} = 10$ cm

$\therefore$ Option (*b*) is correct.

10. (*c*) In the given figure, ABC is an equilateral triangle with AB = BC = CA = $3a$

$AD \perp BC \Rightarrow BD = DC = \dfrac{3a}{2}$



In rt. ΔABD, we have

$$AB^2 = AD^2 + BD^2$$

$$AD^2 = AB^2 - BD^2$$

$$= (3a)^2 - \left(\frac{3a}{2}\right)^2$$

$$= 9a^2 - \frac{9a^2}{4}$$

$$= \frac{36a^2 - 9a^2}{4}$$

$$= \frac{27a^2}{4}$$

$$AD = \frac{3\sqrt{3}a}{2}$$

$\therefore$ Option (*c*) is correct.

11. The given pair of equations are

$$\lambda x + 3y = 7$$
$$2x + 6y = 14$$

$\therefore$ $$\frac{a_1}{a_2} = \frac{\lambda}{2}, \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{7}{14} = \frac{1}{2}$$

For infinitely many solutions, we should have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow$ $$\frac{\lambda}{2} = \frac{3}{6} = \frac{7}{14}$$

$\Rightarrow$ $$\frac{\lambda}{2} = \frac{1}{2} \Rightarrow \lambda = 1$$

$$\lambda = 1$$

The value of λ is equal to 1

12. Mean = 45, mode = 60

Using empirical formula, we have

$\Rightarrow$ $$\text{Mode} = 3 \text{ Median} - 2 \text{ Mean}$$

$\Rightarrow$ $$60 = 3 \text{ Median} - 2 \times 45$$

$\Rightarrow$ $$60 + 90 = 3 \text{ Median}$$

$\Rightarrow$ $$\text{Median} = \frac{150}{3} = 50$$

$\therefore$ Median is 50.

13. Equilateral

OR

Equal

14. Let r and R be the inner and outer radii of the track.

$\therefore$ $$2\pi r = 88 \text{ m and } 2\pi R = 110 \text{ m}$$

$\Rightarrow$ $$r = \frac{88}{2 \times 22} \times 7$$

$$r = 14 \text{ m}$$

$\Rightarrow$ $$R = \frac{110 \times 7}{2 \times 22}$$

$$R = 17.5 \text{ m}$$

$\therefore$ $$\text{Width of track} = R - r$$
$$= (17.5 - 14)\text{m}$$

$\therefore$ $$\text{Width of track} = 3.5 \text{ m}$$

15. $\sec\theta - \cos\theta = 2$

Squaring both sides, we get

$$\sec^2\theta + \cos^2\theta - 2 = 4$$
$$\sec^2\theta + \cos^2\theta = 6$$

Again squaring both sides, we get

$$(\sec^2\theta)^2 + (\cos^2\theta)^2 + 2 = 36$$

$$\therefore \quad \sec^4\theta + \cos^4\theta = 34$$

16. $\cos\theta = \frac{2}{3} \Rightarrow \sec\theta = \frac{3}{2}$

$$\Rightarrow \quad \tan\theta = \sqrt{\sec^2\theta - 1}$$

$$\tan\theta = \sqrt{\left(\frac{3}{2}\right)^2 - 1} = \sqrt{\frac{9}{4} - 1} = \sqrt{\frac{5}{4}}$$

$$\tan\theta = \frac{\sqrt{5}}{2}$$

$$2\sec^2\theta + 2\tan^2\theta - 7 = 2\left(\frac{3}{2}\right)^2 + 2\left(\frac{\sqrt{5}}{2}\right)^2 - 7 = 2 \times \frac{9}{4} + 2\left(\frac{5}{4}\right) - 7 = 0$$

Hence $2\sec^2\theta + 2\tan^2\theta - 7 = 0$

17. Let $\Delta ABC \sim \Delta DEF$

then $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ [Using B.P.T.]

$$\frac{AB}{DE} = \frac{AB + BC + AC}{DE + EF + DF}$$

$$\frac{AB}{DE} = \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF}$$

$$\frac{9}{DE} = \frac{25}{15}$$

$$\Rightarrow \quad DE = \frac{15 \times 9}{25}$$

$$DE = \frac{27}{5}$$

Hence, $DE = 5.4$ cm

18. Height of cylinder (h) = 14 cm

Let radius of cylinder = r cm

$\therefore$ Curved surface area = $2\pi rh$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r \times 14 = 176$$

$$r = \frac{176 \times 7}{2 \times 22 \times 14}$$

$$r = 2 \text{ cm}$$

$\therefore$ diameter of cylinder is $2r = 2 \times 2 = 4$ cm

19. Let P(x, 0) be the point on x-axis which is equidistant from A(2, – 5) and B(– 2, 9)

Given $AP = BP \Rightarrow AP^2 = BP^2$

$$\Rightarrow \quad (x - 2)^2 + (0 + 5)^2 = (x + 2)^2 + (0 - 9)^2$$

$$\Rightarrow \quad x^2 + 4 - 4x + 25 = x^2 + 4 + 4x + 81$$

$$\Rightarrow \quad 8x = -56 \Rightarrow x = -7$$

Hence point (–7, 0) is the point on x-axis which is equidistant from two points (2, – 5) and (–2, 9).

20. AC = 6 cm, BC = 8 cm

ABC is a right triangle with $\angle C = 90°$

$$AC^2 + BC^2 = AB^2$$

$$\Rightarrow \quad (6)^2 + (8)^2 = AB^2$$

$$\Rightarrow \quad AB^2 = 100 \Rightarrow AB = 10 \text{ cm}$$

$\therefore$ Radius of the circle = 5 cm

Area of circle = $\pi r^2 = \dfrac{22}{7} \times 5 \times 5 = \dfrac{550}{7}$ cm^2

Area of right triangle ABC = $\dfrac{1}{2} \times 6 \times 8 = 24$ cm^2

$\therefore$ Area of shaded region = $\left(\dfrac{550}{7} - 24\right)$ cm^2 = (78.57 – 24) cm^2 = 54.57 cm^2

OR

Circumference of circle = $2\pi r = 44$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 44$$

$$\Rightarrow \quad r = \frac{44 \times 7}{2 \times 22}$$

$$r = 7 \text{ cm}$$

Area of quadrant = $\dfrac{1}{4}\pi r^2 = \dfrac{1}{4} \times \dfrac{22}{7} \times 7 \times 7 = 38.5$ cm^2

21. Three numbers are 441, 567 and 693.

Let us first find HCF of two numbers 441 and 567

using Euclid's division algorithm,

Step 1: 567 = 441 × 1 + 126

Step 2: 441 = 126 × 3 + 63

Step 3: 126 = 63 × 2 + 0

$\therefore$ HCF (567, 441) = 63 ...(*i*)

Now we find the HCF of 693 and 63

Step 1: 693 = 63 × 11 + 0

$\therefore$ HCF (693, 63) = 63 ...(*ii*)

From (*i*) and (*ii*) we can write,

HCF (441, 567, 693) = 63

OR

We have $\dfrac{257}{5000} = \dfrac{p}{q}$

$$q = 5000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5 = 2^3 \times 5^4$$

$$\therefore \quad \frac{257}{2^3 \times 5^4} = \frac{257 \times 2}{2^4 \times 5^4} = \frac{514}{10^4} = 0.0514$$

22. Suppose Vinay's car overtakes Raj's car after t hours. Therefore, two cars travels the same distance in t hours.

Distance travelled by Raj's car in t hours = 10 t km

Distance travelled by Vinay's car in t hours = sum of t terms of an A.P. with first term 8 and common difference $\frac{1}{2}$

$\therefore \quad a = 8, d = \frac{1}{2}$

$$= \frac{t}{2}\left[2 \times 8 + (t-1) \times \frac{1}{2}\right]$$

$$= \frac{t}{2}\left[\frac{32 + t - 1}{2}\right]$$

$$= \frac{t(t+31)}{4}$$

When Vinay's car overtakes the Raj's car, we have

$$10t = \frac{t(t+31)}{4}$$

$$40t = t^2 + 31t$$

$$\Rightarrow \quad t^2 + 31t - 40t = 0$$

$$\Rightarrow \quad t^2 - 9t = 0$$

$$\Rightarrow \quad t(t-9) = 0$$

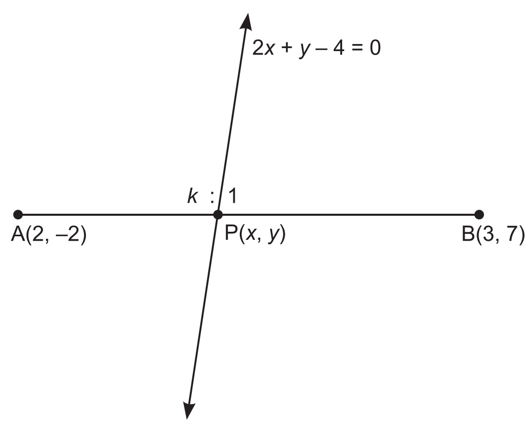
$$\Rightarrow \quad t = 0 \text{ or } t = 9$$

Rejecting $t = 0$, $[\because t \neq 0]$

$$\therefore \quad t = 9$$

Thus, Vinay's car will overtake Raj's car in 9 hours.

23. Let the line $2x + y - 4 = 0$ divides the line segment AB at point P(x, y) in the ratio $k : 1$



Using section formula

x-co-ordinate of P

$$x = \frac{3 \times k + 1 \times 2}{k+1} \Rightarrow x = \frac{3k+2}{k+1}$$

y-coordinate of P

$$y = \frac{7 \times k + 1 \times -2}{k+1} \Rightarrow y = \frac{7k-2}{k+1}$$

$\therefore$ Coordinates of P are $\left[\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right]$

Since P lies on the line $2x + y - 4 = 0$, so it must satisfy the equation $2x + y - 4 = 0$

$$\therefore \quad 2\left(\frac{3k+2}{k+1}\right) + \left(\frac{7k-2}{k+1}\right) - 4 = 0$$

$$6k + 4 + 7k - 2 - 4k - 4 = 0$$

$$9k - 2 = 0$$

$$k = \frac{2}{9}$$

$\therefore$ The required ratio is 2 : 9

24. The given distribution is not continuous so we first change it to a continuous distribution

Age (in years)	**Number of patients**
9.5 – 19.5	19
19.5 – 29.5	$21 \rightarrow f_0$
29.5 – 39.5	$27 \rightarrow f_1$
39.5 – 49.5	$21 \rightarrow f_2$
49.5 – 59.5	22
59.5 – 69.5	20

Since highest frequency is 27

$\therefore$ Modal class is 29.5 – 39.5

$\therefore$ $l = 29.5, f_1 = 27, f_0 = 21, f_2 = 21, h = 10$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 29.5 + \left(\frac{27 - 21}{2 \times 27 - 21 - 21}\right) \times 10 = 29.5 + \frac{60}{12} = 29.5 + 5 = 34.5$$

Hence, the modal age is 34.5 years.

OR

x_i	f_i	$f_i x_i$
3	6	18
5	8	40
7	15	105
9	f	$9f$
11	8	88
13	4	52
Total	$\Sigma f_i = 41 + f$	$\Sigma(f_i x_i) = 303 + 9f$

Here $\Sigma f_i = 41 + f, \Sigma(f_i x_i) = 303 + 9f$

$$\bar{x} = \frac{\sum_{i=1}^{n}(f_i x_i)}{\sum_{i=1}^{n} f_i}$$

$$7.68 = \frac{303 + 9f}{41 + f}$$

$$\frac{768}{100} = \frac{303 + 9f}{41 + f}$$

$$31488 + 768f = 30300 + 900f$$

$\Rightarrow$ $$132f = 1188$$

$$f = \frac{1188}{132}$$

$$f = 9$$

$\therefore$ The missing frequency $f = 9$

25. L.HS. $= \dfrac{\sec^2(90° - \theta) - \cot^2\theta}{2(\sin^2 25° + \sin^2 65°)} - \dfrac{2\cos^2 60° \tan^2 28° \tan^2 62°}{3(\sec^2 43° - \cot^2 47°)}$

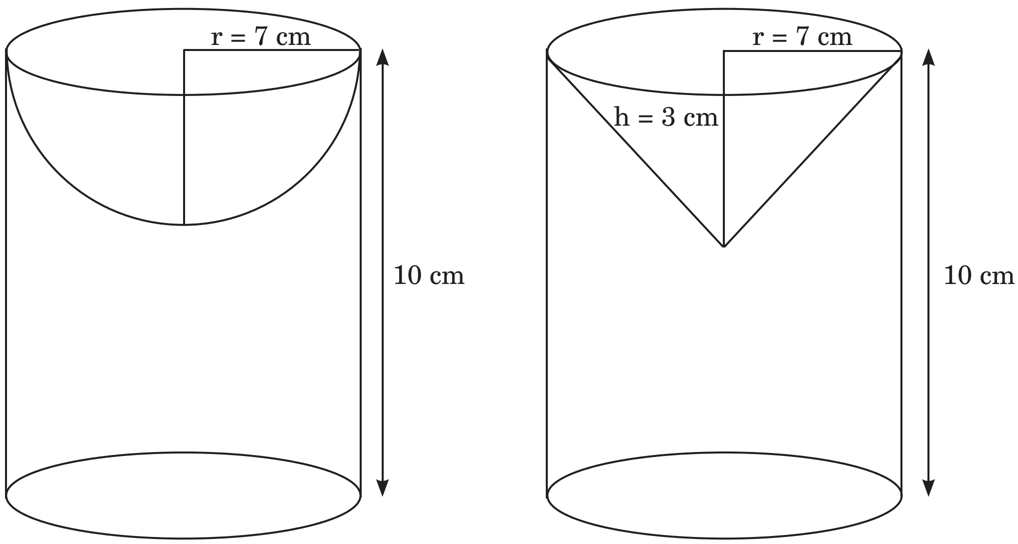
$$= \frac{\text{cosec}^2\theta - \cot^2\theta}{2[\sin^2 25° + \sin^2(90° - 25°)]} - \frac{2 \times \left(\frac{1}{2}\right)^2 \times \tan^2 28° \tan^2[90° - 28°]}{3[\sec^2 43° - \cot^2(90° - 43°)]}$$

$$= \frac{(\text{cosec}^2\theta - \cot^2\theta)}{2[\sin^2 25° + \cos^2 25°]} - \frac{\frac{1}{2}\tan^2 28° \cot^2 28°}{3[\sec^2 43° - \tan^2 43°]}$$

$$= \frac{1}{2(1)} - \frac{\frac{1}{2}(1)}{3(1)} = \frac{1}{2} - \frac{1}{6} = \frac{1}{3} = \text{RHS}$$

26. Height of cylindrical vessel (h) = 10 cm

Radius (r) = 7 cm



Article made by Rohit Article made by Abhishek

Volume of wooden hemispherical article made by Rohit $= \frac{2}{3}\pi r^3$

$$= \left(\frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7\right) \text{cm}^3 = \frac{2156}{3} \text{ cm}^3 = 718.67 \text{ cm}^3$$

Article made by Abhishek

Here r = 7 cm, h = 3 cm

Volume of wooden conical article made by Abhishek $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 3 = 154 \text{ cm}^3$$

Volume of article made by Rohit is more than volume of article made by Abhishek.

27. Here $f(x) = x^3 + 2x^2 + 4x + b$, $g(x) = x + 1$

$q(x) = x^2 + ax + 3$, $r(x) = -3 + 2b$

Using $f(x) = g(x).q(x) + r(x)$, we get

$$\begin{aligned} x^3 + 2x^2 + 4x + b &= (x + 1)(x^2 + ax + 3) + (-3 + 2b) \\ &= x^3 + ax^2 + 3x + x^2 + ax + 3 - 3 + 2b \\ &= x^3 + (a + 1)x^2 + (a + 3)x + 2b \end{aligned}$$

Comparing the coefficients of like terms, we get

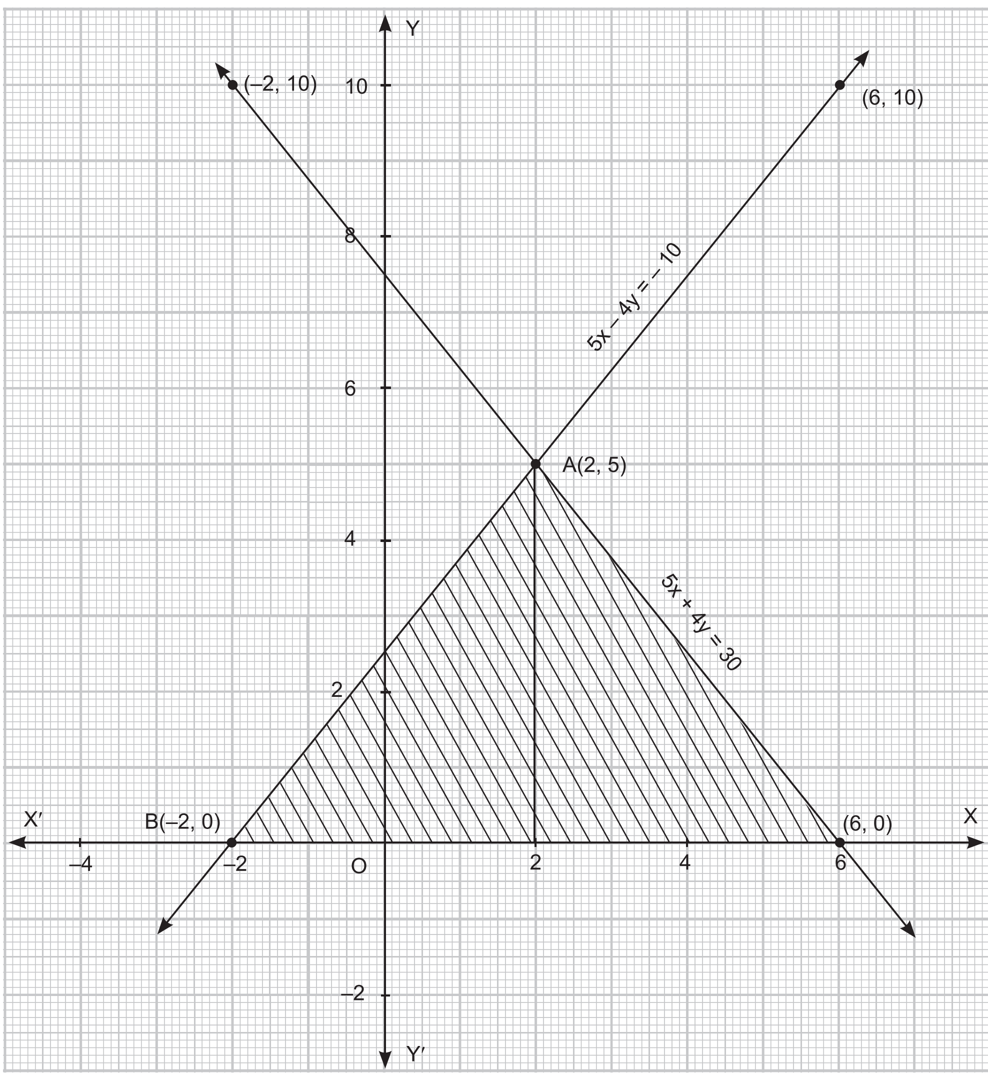
$2 = a + 1$, $4 = a + 3$, $b = 2b$

$\Rightarrow a = 1, b = 0$

28. The given pair of linear equations are

$$5x - 4y = -10 \quad ...(i)$$

and $$5x + 4y = 30 \quad ...(ii)$$



The equation $5x - 4y = -10$ has the following solution:

x	2	–2	6
$y = \frac{5x+10}{4}$	5	0	10

Plot these points on graph,

The equation $5x + 4y = 30$ has the following solution:

x	2	6	–2
$y = \frac{30-5x}{4}$	5	0	10

Plot these points on graph

Hence, from the graph the solution of the pair of lines is $x = 2$ and $y = 5$

Thus, the ΔABC is formed with the vertices A(2, 5) B(– 2, 0) and C(6, 0)

OR

Let monthly incomes of A and B be ₹ $8x$ and ₹ $7x$ respectively and their expenditure be ₹ $19y$ and ₹ $16y$ respectively.

$\therefore \quad 8x - 19y = 2500 \quad ...(i)$

$7x - 16y = 2500 \quad ...(ii)$

$\therefore \quad 8x = 19y + 2500$

or $\quad x = \frac{1}{8}(19y + 2500) \quad ...(iii)$

Substituting in (*ii*), we get

$$7 \times \frac{1}{8}[19y + 2500] - 16y = 2500$$

$$133y + 17500 - 128y = 20,000$$

$$5y = 20,000 - 17500$$

$$5y = 2500$$

$$\therefore \quad y = 500$$

Substituting in (*iii*)

$$x = \frac{1}{8}(19 \times 500 + 2500)$$

$$x = \frac{1}{8}(9500 + 2500)$$

$$x = \frac{12000}{8} = 1500$$

Hence, the incomes of A and B are ₹ (1500 × 8) and ₹ (1500 × 7) i.e. ₹ 12000 and ₹ 10500

29. The given AP is – 11, –7, –3, ... , 49.

Here $a = -11$, $d = 4$, $a_n = 49$

$$a_n = a + (n-1)d = 49$$

$$\Rightarrow \quad -11 + (n-1)\cdot 4 = 49$$

$$\Rightarrow \quad -11 + 4n - 4 = 49$$

$\Rightarrow \quad 4n = 49 + 15$

$\Rightarrow \quad 4n = 64$

$\Rightarrow \quad n = 16$

Since $n = 16$ is an even number, so there are two middle most terms, 8th and 9th

$\therefore \quad a_8 = a + 7d = -11 + 7(4) = 17$

and $a_9 = a + 8d = -11 + 8(4) = 21$

Hence, two middle most terms are 17 and 21.

OR

Let a be the first term and d be the common difference of an A.P.

Given: $a_8 = \frac{1}{2}a_2$

$\Rightarrow \quad a + 7d = \frac{1}{2}(a + d)$

$\Rightarrow \quad 2a + 14d = a + d$

$a = -13d \quad ...(i)$

and $\quad a_{11} = \frac{1}{3}a_4 + 1$

$\Rightarrow \quad a + 10d = \frac{1}{3}(a + 3d) + 1$

$\Rightarrow \quad 3a + 30d = a + 3d + 3$

$2a = -27d + 3 \quad ...(ii)$

Put $a = -13d$ in (ii)

$\Rightarrow \quad 2(-13d) = -27d + 3$

$\Rightarrow \quad -26d = -27d + 3$

$\Rightarrow \quad d = 3$

and $\quad a = -13(3) = -39$

$\therefore \quad a_{15} = a + 14d = -39 + 14(3) = 3$

Hence, $a_{15} = 3$

30. (*i*) Distance of point A(2, – 4) from P(3, 8)

$$\begin{aligned} AP &= \sqrt{(3-2)^2 + (8+4)^2} \\ &= \sqrt{(1)^2 + (12)^2} \\ &= \sqrt{1+144} \\ &= \sqrt{145} \end{aligned}$$

Distance of point A(2, – 4) from Q(–10, – 5)

$$\begin{aligned} AQ &= \sqrt{(-10-2)^2 + [-5-(-4)]^2} \\ &= \sqrt{(-12)^2 + (-5+4)^2} \\ &= \sqrt{144 + (-1)^2} \\ &= \sqrt{144+1} \\ &= \sqrt{145} \end{aligned}$$

$$AP = AQ$$

(*ii*) Yes, hometowns are equidistant from their school.

(*iii*) Area of triangle formed by A(2, – 4), P(3, 8), Q(–10, – 5)

$$\text{area of } \Delta APQ = \frac{1}{2}|2(8+5)+3(-5+4)-10(-4-8)|$$

$$= \frac{1}{2}|2(13)+3(-1)-10(-12)|$$

$$= \frac{1}{2}|26-3+120|$$

$$= \frac{1}{2}|143|$$

$$= \frac{143}{2} \text{ sq. units} = 71.5 \text{ sq. units}$$

31.

$$\text{L.H.S.} = \sqrt{\sec^2 A + \text{cosec}^2 A}$$

$$= \sqrt{\frac{1}{\cos^2 A} + \frac{1}{\sin^2 A}}$$

$$= \sqrt{\frac{\sin^2 A + \cos^2 A}{\sin^2 A \cos^2 A}}$$

$$= \frac{1}{\sqrt{\sin^2 A \cos^2 A}} \qquad [\because \sin^2 A + \cos^2 A = 1]$$

$$= \frac{1}{\sin A \cdot \cos A}$$

$$= \frac{\sin^2 A + \cos^2 A}{\sin A \cos A}$$

$$= \frac{\sin^2 A}{\sin A \cdot \cos A} + \frac{\cos^2 A}{\sin A \cdot \cos A} = \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

$$= \tan A + \cot A = \text{R.H.S.}$$

Hence, $\sqrt{\sec^2 A + \text{cosec}^2 A} = \tan A + \cot A$

OR

$$\text{LHS} = \frac{1}{\sec\theta + 1} + \frac{1}{\sec\theta - 1}$$

$$= \frac{\sec\theta - 1 + \sec\theta + 1}{(\sec\theta + 1)(\sec\theta - 1)}$$

$$= \frac{2\sec\theta}{\sec^2\theta - 1}$$

$$= \frac{2\sec\theta}{\tan^2\theta}$$

$$= \frac{2 \times \frac{1}{\cos\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= 2 \times \frac{1}{\cos\theta} \times \frac{\cos^2\theta}{\sin^2\theta}$$

$$= 2 \cdot \frac{1}{\sin\theta} \cdot \frac{\cos\theta}{\sin\theta}$$

$$= 2\text{cosec}\,\theta \cdot \cot\theta = \text{R.H.S.}$$

32. (*i*) x-coordinate of point of intersection of less than ogive and more than ogive is the median.

$\therefore$ In the given graph median is 60

(*ii*) Given: median = 60, Mean = 59.30

By empirical relationship,

$$\text{Mode} = 3\text{ Median} - 2\text{ Mean}$$

$$\text{Mode} = 3 \times 60 - 2 \times 59.30$$

$$= 180 - 118.60$$

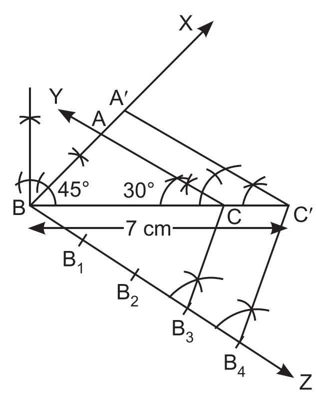
$$\therefore \quad \text{Mode} = 61.40$$

(*iii*)

Class	**Frequency**
20 – 30	8
30 – 40	12
40 – 50	24
50 – 60	6
60 – 70	10
70 – 80	15
80 – 90	25

33. Steps of Construction:

Here $\angle C = 180° - 45° - 105° = 30°$



Step 1: Draw a line segment BC = 7 cm

Step 2: At B, construct $\angle XBC = 45°$ and at C construct $\angle BCY = 30°$ to intersect at A. Then ΔBAC is the first triangle.

Step 3: Through B, construct $\angle ZBC$ an acute angle.

Step 4: On BZ, mark points B_1, B_2, B_3, B_4 such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$

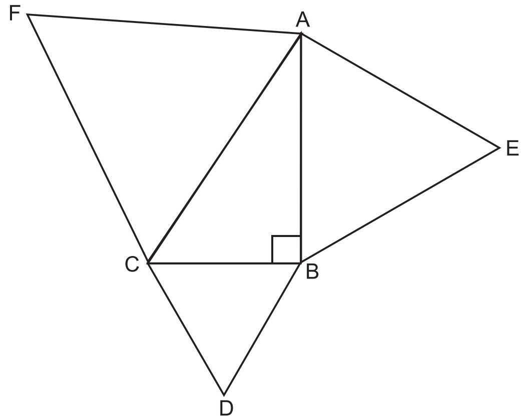
Step 5: Join B_3C and from B_4 draw $B_4C' \,||\, B_3C$ to meet BC produced at C′.

Step 6: Through C′, draw line parallel to CA to meet BA produced at A′.

Then, $\Delta A'BC'$ is the required triangle.

34. **Given:** In the given figure, ABC is a right triangle with $\angle B = 90°$

ΔABE, ΔBCD and ΔACF are equilateral triangles on sides AB, BC and AC.



To prove: $\text{ar}(\Delta ACF) = \text{ar}(\Delta ABE) + \text{ar}(\Delta BCD)$

Proof: Now, $$\text{area}(\Delta ABE) = \frac{\sqrt{3}}{4}(AB)^2 \quad ...(i)$$

$$\text{area}(\Delta BCD) = \frac{\sqrt{3}}{4}(BC)^2 \quad ...(ii)$$

and $$\text{area}(\Delta ACF) = \frac{\sqrt{3}}{4}(AC)^2 \quad ...(iii)$$

Adding (i) & (ii), we get

$$\text{ar}(\Delta ABE) + \text{ar}(\Delta BCD) = \frac{\sqrt{3}}{4}(AB)^2 + \frac{\sqrt{3}}{4}(AC)^2$$

$$= \frac{\sqrt{3}}{4}[(AB)^2 + (BC)^2]$$

$$= \frac{\sqrt{3}}{4}AC^2 \quad [\because AB^2 + BC^2 = AC^2]$$

$$= \text{ar}(\Delta ACF)$$

Hence, area of equilateral triangle drawn on hypotenuse AC = sum of areas of the equilateral triangles drawn on the other two sides AB and BC.

35. Let us assume that $\frac{1}{5+\sqrt{3}}$ is rational

Now $\frac{1}{5+\sqrt{3}} \times \frac{5-\sqrt{3}}{5-\sqrt{3}} = \frac{5-\sqrt{3}}{22} = \frac{a}{b}$, where, $b \neq 0$, a & b are integers

$$\Rightarrow \quad \frac{5-\sqrt{3}}{22} = \frac{a}{b}$$

$$\Rightarrow \quad 5-\sqrt{3} = \frac{22a}{b}$$

$$\Rightarrow \quad \sqrt{3} = 5 - \frac{22a}{b}$$

$$\sqrt{3} = \frac{5b-22a}{b}$$

Since right hand side is rational, this leads to contradict the fact that $\sqrt{3}$ is irrational.

$\therefore$ Our supposition is worng

Thus $\frac{1}{5+\sqrt{3}}$ is an irrational.

36. The pair of equations is

$$\frac{b}{a}x + \frac{a}{b}y = a^2 + b^2$$
$$x + y = 2ab$$

The given equations can be written as

$$b^2x + a^2y - (a^3b + ab^3) = 0$$
$$x + y - 2ab = 0$$

We can write the coefficients as

	x	y	1
a^2	$-(a^3b+ab^3)$	b^2	a^2
1	$-2ab$	1	1

$$\Rightarrow \quad \frac{x}{-2a^3b + 1(a^3b + ab^3)} = \frac{y}{-1(a^3b + ab^3) + 2ab^3} = \frac{1}{b^2 - a^2}$$

$$\Rightarrow \quad \frac{x}{-a^3b + ab^3} = \frac{y}{-a^3b + ab^3} = \frac{1}{b^2 - a^2}$$

$$\Rightarrow \quad \frac{x}{ab(b^2 - a^2)} = \frac{y}{ab(b^2 - a^2)} = \frac{1}{b^2 - a^2}$$

or $$x = \frac{ab(b^2 - a^2)}{b^2 - a^2}, y = \frac{ab(b^2 - a^2)}{b^2 - a^2}$$

$$\Rightarrow \quad x = ab, y = ab$$

Hence $x = ab$ and $y = ab$ is the required solution.

OR

Let units digit be x and tens digit be y.

Then, the number is = $10y + x$

According to question

$$(10y + x) + (10x + y) = 132$$
$$\Rightarrow \quad 11x + 11y = 132$$
$$\Rightarrow \quad x + y = 12 \quad \text{...}(i)$$
and $$(10y + x) + 12 = 5(x + y)$$
$$\Rightarrow \quad 10y + x + 12 = 5x + 5y$$
$$\Rightarrow \quad 4x - 5y = 12 \quad \text{...}(ii)$$

Multiplying equation (*i*) by 5 and add it to equation (*ii*), we get

$$4x - 5y + 5x + 5y = 12 + 60$$

$$\Rightarrow \quad 9x = 72$$

$$\Rightarrow \quad x = 8$$

Substituting in (*i*) we get

$$8 + y = 12$$

$$y = 4$$

Hence, the number is 48.

37. Elementary events associated to the random experiment of throwing two dice are:

$$\begin{bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{bmatrix}$$

$\therefore$ Total number of elementary events = 6 × 6 = 36

(*i*) Let A be event of getting a total of at least 9 i.e. 9, 10, 11, 12. Then elementary events favourable to A are:

(3, 6), (4, 5), (5, 4), (6, 3), (4, 6), (5, 5), (6, 4), (5, 6), (6, 5), (6, 6)

Favourable number of elementary events = 10

Hence, required probability = $\frac{10}{36} = \frac{5}{18}$

(*ii*) Let A be the event of getting a multiple of 2 on one die and a multiple of 3 on the other.

Then elementary event favourable to A are (2, 3), (2, 6), (4, 3), (4, 6), (6, 3), (6, 6), (3, 2), (3, 4), (3, 6), (6, 2), (6, 4)

$\therefore$ Favourable number of elementary events = 11

Hence required probability = $\frac{11}{36}$

(*iii*) Let A be event of getting a same number on both the dice. Then elementary events favourable to A are:

(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

$\therefore$ Favourable number of elementary events = 6

Hence required probability = $\frac{6}{36} = \frac{1}{6}$

(*iv*) Let A be the event of getting the sum as a prime number i.e. 2, 3, 5, 7, 11. Elementary events favourable to event A are:

(1, 1), (1, 2), (2, 1), (1, 4), (2, 3), (3, 2) (4, 1), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1), (5, 6), (6, 5).

$\therefore$ Favourable number of elementary events = 15

Hence, required probability = $\frac{15}{36} = \frac{5}{12}$

OR

Total possible outcomes = 99

(*i*) Let A be event of getting a number divisible by 3 and 5 is divisible by 15.

Then, elementary events favourable to A are:

15, 30, 45, 60, 75, 90

$\therefore$ Favourable number of elementary event = 6

Hence, required probability = $\frac{6}{99} = \frac{2}{33}$

(*ii*) The probability of numbers not divisible by 3 and 5 are

$= 1 -$ (Probability of numbers divisible by 3 and 5)

$= 1 - \frac{2}{33} = \frac{31}{33}$

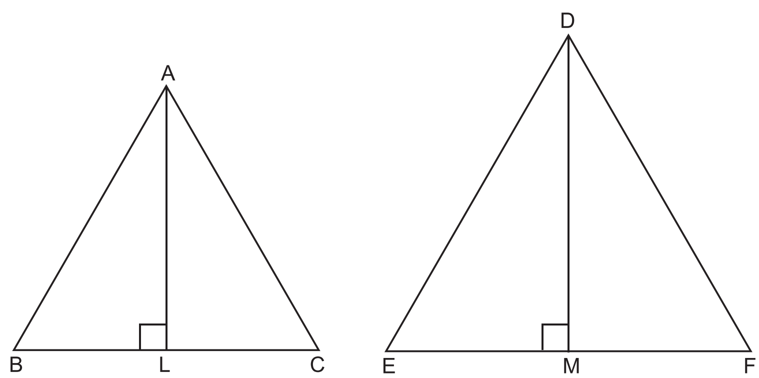
(*iii*) Let A denote the event of getting a perfect square. Then, elementary events favourable to A are: 1, 4, 9, 16, 25, 36, 49, 64, 81.

$\therefore$ Favourable numbers of elementary events = 9

Hence, required probability = $\frac{9}{99} = \frac{1}{11}$

38. Given: Two triangles ABC and DEF such that $\Delta ABC \sim \Delta DEF$

$AL \perp BC$ and $DM \perp EF$



To prove: $\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$

Proof: Given $\Delta ABC \sim \Delta DEF$

$\therefore \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ ...(*i*)

and $\angle A = \angle D, \angle B = \angle E$ and $\angle C = \angle F$

In ΔABL and ΔDEM,

$\angle B = \angle E$

and $\angle L = \angle M = 90°$

$\therefore \quad \Delta ABL \sim \Delta DEM$ (AA similarity)

$\therefore \quad \frac{AB}{DE} = \frac{AL}{DM}$ ...(*ii*)

From (i) and (ii), we have

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{AL}{DM} \quad ...(iii)$$

Now $$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{\frac{1}{2} \times BC \times AL}{\frac{1}{2} \times EF \times DM}$$

$$= \frac{BC}{EF} \times \frac{AL}{DM}$$

$$= \frac{BC}{EF} \times \frac{BC}{EF} \quad \text{[Using (iii)]}$$

$\therefore$ $$\frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)} = \left(\frac{BC}{EF}\right)^2 \quad ...(iv)$$

$\therefore$ From (iii) and (vi),

$$\frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$$

OR

(i) In ΔABC, we have

$AF = AE$ ...(i) [tangent from external points to a circle are equal]

$BF = BD$ ...(ii)

and $CD = CE$ ...(iii)

Adding (i), (ii) and (iii), we get

$$AF + BF + CD = AE + BD + CE$$

$$(AF + BF) + CD = (AE + CE) + BD$$

$$AB + CD = AC + BD$$

(ii) $$\text{Area }(\Delta ABC) = ar(\Delta AOB) + ar(\Delta BOC) + ar(\Delta AOC)$$

$$= \frac{1}{2} \times AB \times OF + \frac{1}{2} \times BC \times OD + \frac{1}{2} \times AC \times OE$$

$$= \frac{1}{2}[(AB \cdot r) + (BC \cdot r) + AC \cdot r]$$

$$= \frac{1}{2} \cdot r(AB + BC + AC)$$

$$= \frac{1}{2}r(\text{Perimeter of } \Delta ABC)$$

39. Let AB be the tower of height h meters. Let C be the initial point of the car and let after 12 minutes the car be at point D.

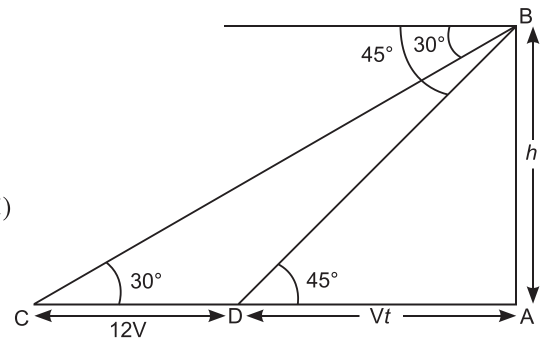
Let speed of the car be V meter per minute.

Then, CD = Distance travelled by car in 12 minute

$CD = 12V$ meter [$\because$ Distance = speed × time]

Suppose the car take t minutes to reach the tower AB from D. Then DA = Vt meters

In ΔDAB, we have



$$\tan 45° = \frac{AB}{AD}$$

$$1 = \frac{h}{Vt}$$

$$\Rightarrow \quad h = Vt \quad ...(i)$$

In ΔCAB, we have

$$\tan 30° = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{Vt + 12V}$$

$$\Rightarrow \quad \sqrt{3}h = Vt + 12V \quad ...(ii)$$

Substituting the value of h from equation (i) in equation (ii), we get

$$\sqrt{3}Vt = Vt + 12V$$

$$\Rightarrow \quad \sqrt{3}t = t + 12$$

$$t(\sqrt{3} - 1) = 12$$

$$t = \frac{12}{\sqrt{3} - 1}$$

$$t = \frac{12}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$= \frac{12(\sqrt{3} + 1)}{3 - 1}$$

$$\therefore \quad t = 6(\sqrt{3} + 1)$$

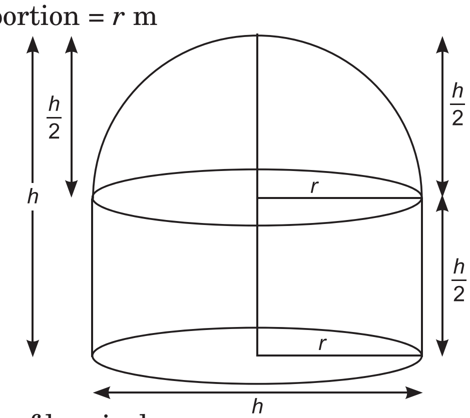
$$= 16.39 \text{ minutes}$$

$$\Rightarrow \quad t = 16 \text{ minutes } 23 \text{ seconds}$$

[$\because$ 0.39 minutes = 0.39 = 0.39 × 60 seconds = 23 second]

Thus, the car will reach the tower from D in 16 minutes and 23 seconds.

40. Let height of Building h m and radius of circular portion = r m



$\therefore$ Diameter = height of building

$\therefore$ $2r = h$

$\therefore$ Radius $r = \frac{h}{2}$ m

$\therefore$ Height of cylindrical portion = $\frac{h}{2}$ m

Volume of building = $41\frac{19}{21}$ m^3 (given)

Volume of Building = Volume of cylinder + Volume of hemisphere

$$= \pi r^2\left(\frac{h}{2}\right) + \frac{2}{3}\pi r^3$$

$$\Rightarrow \quad \pi\left[\frac{h}{2}\right]^2 \cdot \left(\frac{h}{2}\right) + \frac{2}{3}\pi\left(\frac{h}{2}\right)^3 = 41\frac{19}{21}$$

$$\pi\frac{h^3}{8}\left[1+\frac{2}{3}\right] = \frac{880}{21}$$

$$\frac{5}{24}\pi h^3 = \frac{880}{21}$$

$$\frac{5}{24} \times \frac{22}{7} \times h^3 = \frac{880}{21}$$

$$h^3 = \frac{880}{21} \times \frac{24 \times 7}{5 \times 22}$$

$$\Rightarrow \quad h^3 = 64$$

$$\Rightarrow \quad h = 4$$

$$\therefore \quad h = 4 \text{ m}$$

$$\therefore \quad r = \frac{h}{2} = 2 \text{ cm}$$

Total surface area of building = curved surface area of cylinder + curved surface area of hemisphere

$$= 2\pi r\left(\frac{h}{2}\right) + 2\pi r^2$$

$$= 2\pi r\left(\frac{h}{2} + r\right)$$

$$= 2 \times \frac{22}{7} \times \left(\frac{4}{2} + 2\right) \times 2$$

$$= \frac{88}{7} \times 4 = 50.29 \text{ m}^2$$

$\therefore$ Surface area of building = 50.29 m^2.